Uncertain Innovation and R&D Network Formation

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Motivation

- Collaboration among firms in innovative activities is now widespread (Hagedoorn, 2002; Powell et al., 2005)

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- Majority of these collaborations are bilateral.
- Understanding collaboration networks and their impact on the industry is important.

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  - Help by lowering R&D costs
  - Help share the risk associated with innovation

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  - Help share the risk associated with innovation

- R&D collaborations do not always lead to an innovation.

- Firms characteristics (*technological, business or product similarity, geographical proximity*) affect innovation success probability

Model features

- Oligopoly setting in which (horizontally related) firms form pair-wise collaborative links with other firms.

- These bilateral links require commitment of resources which are used for R&D.

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- Oligopoly setting in which (horizontally related) firms form pair-wise collaborative links with other firms.
- These bilateral links require commitment of resources which are used for R&D.
- Successful R&D leads to innovations.
- If an innovation occurs it results in lower production costs for the firms involved in the link.

Related literature

- Goyal and Moraga-Gonzales (2001); Goyal, Moraga-Gonzales and Konovalov (2008):

Analyze effort in R&D – the interaction between effort of a firm in collaboration link and their effort in other R&D projects in equilibrium.

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Analyze effort in R&D – the interaction between effort of a firm in collaboration link and their effort in other R&D projects.

- Konig *et al*. (2012)

Direct and indirect network spillovers matter, so every firm in a component has the same payoff. No market structure and strategic element is missing.

Related literature

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- Westbrock (2010) and Billand et al. (2015)

Similar setting, but for welfare comparisons.

Structure of the Game

- Stage 1: Firms simultaneously chose the collaborative R&D links they wish to form.
  - These choices induce a network g
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- Stage 2: Firms play simultaneous oligopoly game

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- Strategy of firm i: $g_i = \{\{g_{ij}\}_{j \in N_{-i}}\}$

Model setup

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- Strategy of firm i: $s_i = \{\{s_{ij}\}_{j \in N_{-i}}\}$
- A link is formed iff $s_{ij} = s_{ji} = 1$
- A strategy profile $s = \{s_1, s_2, \ldots, s_n\}$ induces a network $g^{[s]}$.

Model setup

- $g(i)$: set of firms with whom firm i has a link.
- $|g(i)|$ - its cardinality

- $\mathcal{N}(g)$: set of firms with at least one link.

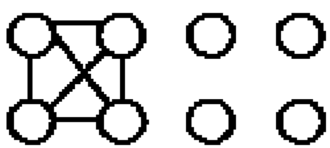
- $g[N']$: sub-network of g defined on $N' \subset N$
- g_{-i}: $g[N \setminus \{i\}]$

- $g+ij$; $g-ij$

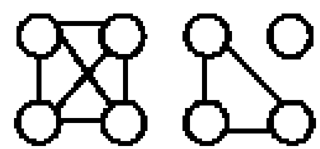
Network architectures

- *Complete and Empty*

- *Group dominant network: $g[\mathcal{N}(g)]$* is complete.



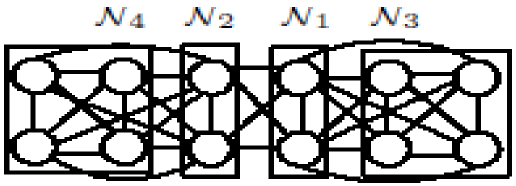
A group-dominant network



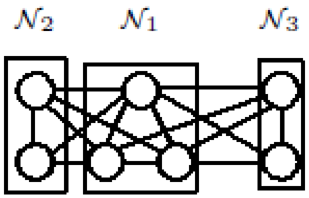
A 2-group-dominant network

Network architectures

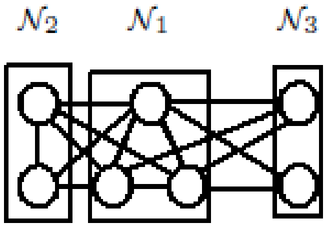
- *2|2 hierarchical network*: Partition g into 4 groups which are all complete and firms in $\mathcal{N}_1$ are linked only to firms in $\mathcal{N}_2 \cup \mathcal{N}_3$ and firms in $\mathcal{N}_2$ are linked only to firms in $\mathcal{N}_4$.



A 2|2-hierarchical network



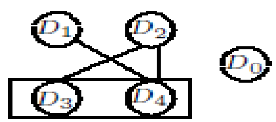
A 1|2-hierarchical network



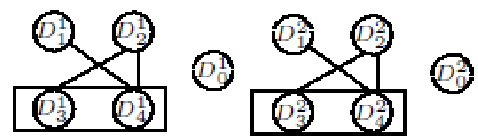
A 1|(1, 1)-hierarchical network

Network architectures

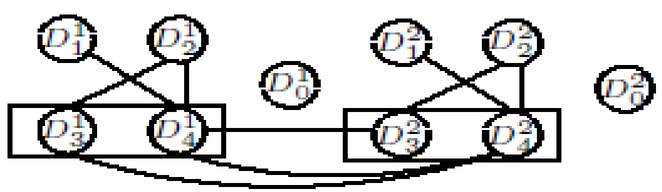
- *Nested split graph:* Have a nested neighborhood structure, i.e., the set of neighbors of each agent is contained in the set of neighbors of each higher degree agent.



A NSG



A 2-NSG



A multi-NSG

Flows/probabilities of innovation

- $\rho_{ij} \in (0,1]$: probablity that the R&D collaboration between i and j yields a successful innovation

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- $\rho_{ij} = \rho_{ji}$ and $B = (\rho_{ij})_{i \in N, j \in N_{-i}}$
  - Probabilities are independent of the network
  - Probabilities are independent of each other
  - These probabilities can be a function of firm characteristics

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  - Probabilities are independent of the network; of each other; and can be a function of firm characteristics

- Each firm has a flow degree:

$$U_i(g) = \sum_{j \in g(i)} \rho_{ij} \text{ and } U(g) = \sum_{ij \in g} \rho_{ij}$$

$$U(g_{-i}) = U(g) - U_i(g)$$

Stability concept

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  1. There is a Nash equilibrium strategy profile which supports *g*.

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- A network g is a pair-wise equilibrium network if the following conditions hold (*Goyal and Joshi*, 2006):

1. There is a Nash equilibrium strategy profile which supports g.

2. For $g_{ij} = 0$,

$$\Pi^*{}_i \ (g + ij) - \Pi^*{}_i \ (g) > 0 \Longrightarrow$$

$$\Pi^*{}_j \ (g + ij) - \Pi^*{}_j \ (g) < 0$$

How innovations affects cost

- Each R&D link requires a fixed investment: $f > 0$

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- Cost function

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Each collaboration is associated with a specific innovation.

How innovations affects cost

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- Cost function

$$c_i(g) = \gamma_0 - \gamma \sum_{j \in g(i)} \rho_{ij} = \gamma_0 - \gamma U_i(g)$$

- A network g induces an expected cost vector for firms:

$$c(g) = (c_1(g), c_2(g), \ldots, c_n(g))$$

Homogeneous Cournot oligopoly

- Demand: $p = \alpha - \sum_{i \in N} q_i$

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- Stage 2 profits:

$$\Pi^*{}_i\ (g) = \left(\frac{\alpha - \gamma_0 + n\gamma U_i(g) - \gamma \sum_{j \in N\{i\}} U_j(g)]}{n+1}\right)^2$$

$$=(a + bU_i(g) - cU(g_{-i}))^2 = \varphi(U_i(g), U(g_{-i}))$$

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Note that marginal profits from a additional link are increasing in the first argument and decreasing in the second.

Homogeneous Cournot oligopoly

- Stage 1 profits: $\prod_i^*(g) = \Pi^*{}_i\ (g) - |g(i)|f$

- CS = 1/2$\left(\frac{n(\alpha-\gamma_0)+2\gamma U(g)}{n+1}\right)^2 = \phi\big(U(g)\big)$

Homogeneous Cournot oligopoly

- Stage 1 profits: $\prod_i^*(g) = \Pi^*{}_i\ (g) - |g(i)|f$
- CS = 1/2$\left(\frac{n(\alpha-\gamma_0)+2\gamma U(g)}{n+1}\right)^2 = \phi\big(U(g)\big)$
- Social welfare: $W(g) = \text{CS} + \prod_i^*(g) = \phi(U(g), |g|)$
- An efficient network is one that maximizes *W(g)*.

Too many probabilities?

- Can we use stylized facts to improve things?
  - Characteristics like geographical, technological or business similarities of firms matter.

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  - Characteristics like geographical, technological or business similarities of firms matter.

The Insider-Outsider model

- Two groups of firms: N^1 and N^2
- Success probability between firms i and j is higher if they belong to the same group.
- Formally, $\rho_{ij} = \rho^I$ if $i, j \in N^t, t \in \{1,2\}$ and ρ^O otherwise, where $\rho^I > \rho^O$.

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- Can we use stylized facts to improve things?
  - Some firms are more aggressive about innovations
  - Firms with greater market share are more likely to be innovators

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- Can we use stylized facts to improve things?
  - Some firms are more aggressive about innovations
  - Greater market share → more likely to be innovators

High and Low innovative firms

- Two groups of firms N^H and N^L with the same cardinality.
- Formally, $\rho_{ij} = \rho^t$ if $i, j \in N^t, t \in \{H, L\}$ and ρ^M otherwise, where $\rho^H > \rho^M > \rho^L$.

Results: Pws-Equilibrium

Proposition 0*: What if links are not costly?*

Suppose $f = 0$. Then a pair-wise equilibrium network is the complete network.

Results: Pws-Equilibrium

Proposition 1: *What does uncertainty imply for link formation?*

Let g be a pair-wise equilibrium network. If $i \in \mathcal{N}^{\rho}$ and $j \in \mathcal{N}^{\rho'}$, with $\rho \geq \rho'$ and $\rho_{ij} > \rho$, then there is a link between i and j in g.

Results: *How probabilities matter…*

***Corollary 1**: One probability*

Let g be a pair-wise equilibrium network. If $\rho_{ij} = \rho$*, then g is a group dominant network.*

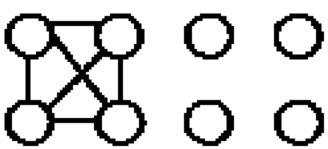
$\Rightarrow$ *Proposition 4.1 of Goyal and Joshi, 2003 is a special case when innovations occur with full certainty.*

$\Rightarrow$ *By continuity the result is also true when probabilities differ but are sufficiently similar.*

Results: *How probabilities matter…*

***Corollary 2**: Two probabilities*

Suppose the assumptions of the I-O framework are satisfied. If g is a non-empty pair-wise equilibrium network, then it is a group dominant network, or a 2-group dominant network or a 2|2 *hierarchical network.*



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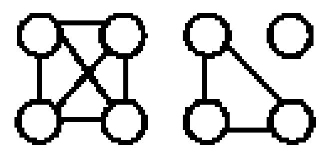


A 2-group-dominant network

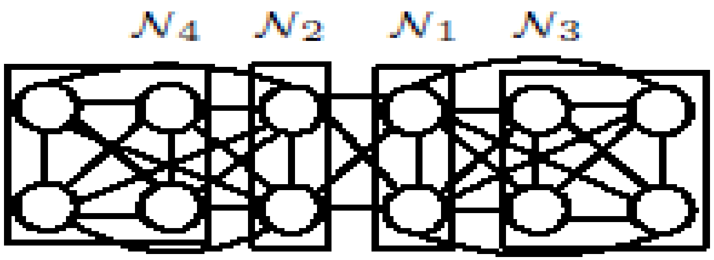
Results: *How probabilities matter…*

***Corollary 3**: Three probabilities*

Suppose the assumptions of the H-L framework are satisfied. If g is a non-empty pair-wise equilibrium network, then it is a group dominant network, or a 2-group dominant network, or a $1|2$ *hierarchical network, or a* $1|(1,1$*) hierarchical network.*

Moreover, (i) firms in N^H *that have formed links are all linked together, and (ii) firms in* N^H *that have formed links with firms in* N^L *are linked with all firms in* N^L *that have formed links.*

Network architectures



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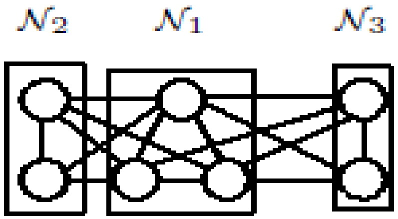
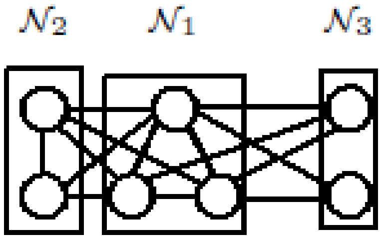


A 1|2-hierarchical network



A 1|(1, 1)-hierarchical network

Results: *How probabilities matter…*

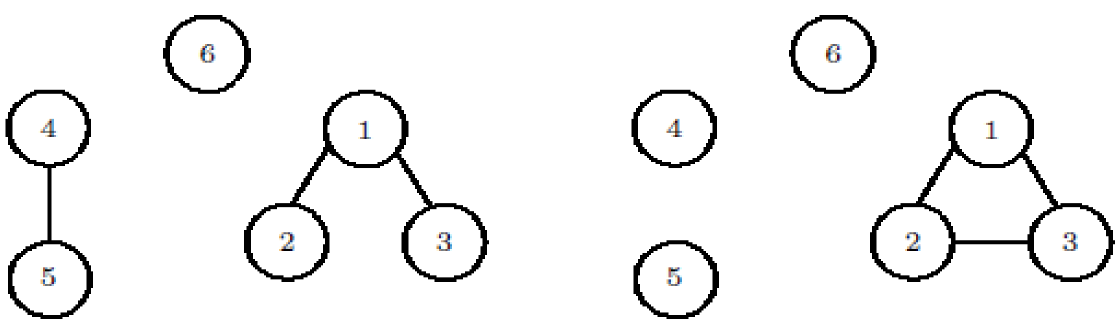
1. *Pairwise equilibrium networks are group dominant networks or variations (hierarchical versions) of those.*

2. *The most valuable links may not be formed in a pairwise stable network.*

Results

***Proposition 2 (Idea)**: Non monotonicity of pw-equilibrium*

There exist situations in which: If the success probabilities of links in a pw-equilibrium increase, then some existing links may be deleted.



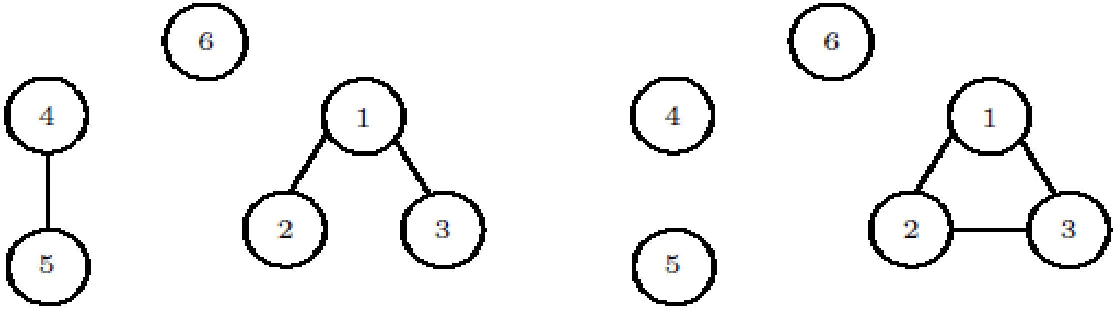
Network g Network g'

Results

Proposition 2 (Idea): *Non monotonicity of pw-equilibrium*

There exist situations in which: If the success probabilities of links in a pw-equilibrium increase, then some existing links may be deleted.

- *Welfare is lower in the new equilibrium.*
- *This has implications for policy.*



Network g Network g'

Results: Welfare

- ***Proposition 3***: Finding efficient networks

Let g be an efficient network that contains a link between firms i *and* j*. If* $c_j \geq c_{j'}$ *and* $\rho_{ij} \leq \rho_{ij'}$*, then there is a link between firms* i *and* j' *in g.*

Results: Welfare

Corollary 4*: One probability*

Suppose that for all $i, j \in N$, $\rho_{ij} = \rho$, *then an efficient network is a NSG.*

Results: Welfare

***Corollary 4**: One probability*

Suppose that for all $i, j \in N$, $\rho_{ij} = \rho$, then an efficient network is a NSG.

***Corollary 5**: Two probabilities*

Suppose the assumptions of the I-O framework are satisfied. If g is a non-empty efficient network, then it is a NSG, or a 2-NSG or a multi-NSG.

Results: Welfare

***Corollary 6**: Three probabilities*

Suppose the assumptions of the H-L framework are satisfied. If g is a non-empty efficient network, then it is a NSG, or a group NSG or a multi-NSG. Moreover if $g[N^H]=\emptyset$*, then* $g[N^L]=\emptyset$*.*

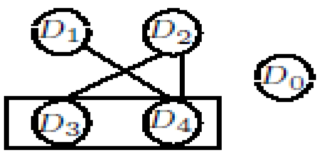
Results: Welfare

***Corollary 6**: Three probabilities*

Suppose the assumptions of the H-L framework are satisfied. If g is a non-empty efficient network, then it is a NSG, or a group NSG or a multi-NSG. Moreover if g[N^H]=Ø, then g[N^L]=Ø.

It is easy to show in the H-L framework that there is a conflict between stability and efficiency.

Nested split graphs



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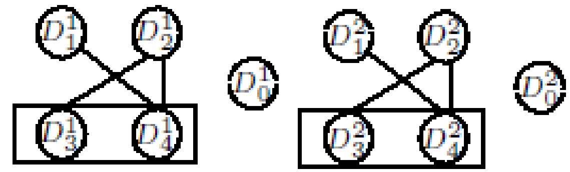
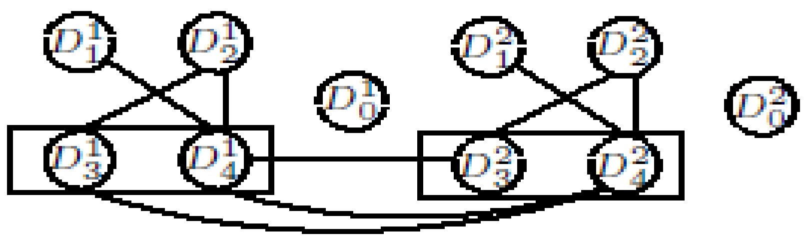


A 2-NSG



A multi-NSG

Results

Proposition 4: *About a larger class of oligopoly games*

Let $\Pi_i(g) = \sigma(U_i(g), U(g_{-i}))$

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***Proposition 4**: About a larger class of oligopoly games*

Let $\Pi_i(g) = \sigma(U_i(g), U(g_{-i}))$

Suppose the payoff function is as shown above and σ is strictly increasing in its first argument, strictly convex and sub-modular. Let g be a pw-equilibrium network and suppose $ij \in L(g)$ *and* $i'j' \notin L(g)$. *Then either* $\mathrm{U}_{ij}^{m}(g) > \mathrm{U}_{i'j'}^{m}(g)$ *or* $\rho_{ij} > \rho_{i'j'}$.

Results

- These results hold for the differentiated Cournot and Bertrand models.
- Analogous version of Proposition 1 (Corollary 7) will exist in this case.
- Non-monotonicity in links (Proposition 2) result will hold
- Conditions for efficient networks are shown (Proposition 5) and an analogous version of Proposition 3 can be shown.

Summing up

- On introducing uncertainty we find that
  - Pws-equilibrium networks are dominant networks or their variations, i.e., Goyal and Joshi (2003) is a special case.
  - Efficient networks are variations of NSG, i.e., Westbrock (2010) and Billand et al. (2015) are special cases.

Summing up

- On introducing uncertainty we find that
  - Pws-equilibrium networks are dominant networks or their variations, i.e., Goyal and Joshi (2003) is a special case.
  - Efficient networks are variations of NSG, i.e., Westbrock (2010) and Billand et al. (2015) are special cases.
- Public policy aimed at increasing innovative activity has to be done carefully.
- Results can be extended to a general class of games.

Two things…

- Goyal and Joshi (2003) use 3 properties to obtain their results:
  1) All links lead to the same reduction in marginal costs
  2) The profit function is convex in own links
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- Uncertainty not only makes the model more realistic, it introduces heterogeneity and relaxes (1)
  - Alters the formal analysis
  - Goyal and Joshi (2003) is a special case

Two things…

- What if firms can absorb innovations at different rates?
  - The same innovation can affect two firms differently.

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$\Rightarrow$ *A "tyranny of the weakest" type situation in equilibrium.*

$\Rightarrow$ *Positive assortative matching in equilibrium.*


THANK
YOU!